Expanding Contour Spring Derivation

$$\textbf{Eqn1} = \frac{\textbf{F[xr]}}{\textbf{Fs[xr]}} == \textbf{yc'[xc]}$$

$$\frac{F[xr]}{Fs[xr]} == yc'[xc]$$

$$\textbf{Eqn2 = Eqn1 /. } \left\{\textbf{F[xr] -> a xr}^2 \textbf{ + b xr + c, Fs[xr] -> 4 k} \left(\textbf{yr[xr]} - \frac{\textbf{x0}}{\textbf{2}} \textbf{ + Rpin}\right) \textbf{ + 2 Fp}\right\}$$

$$\frac{c + b\,xr + a\,xr^2}{2\,Fp + 4\,k\,(Rpin - \frac{x0}{2} + yr[xr])} == yc'[xc]$$

$$\textbf{Eqn3 = Eqn2 /. \{yr[xr]} \to \textbf{yc[xc] + R Cos[}\varphi\textbf{[xr]]\}}$$

$$\frac{c + b\,xr + a\,xr^2}{2\,Fp + 4\,k\,(Rpin - \frac{x0}{2} + R\,Cos[\varphi[xr]] + yc[xc])} == yc'[xc]$$

$$\textbf{Eqn4 = (Numerator[Eqn3[[1]]] /. \{xr} \to \textbf{xc - R Sin[}\varphi\textbf{[xr]]\}) == Eqn3[[2]] Denominator[Eqn3[[1]]]}$$

$$c + b\,(xc - R\,Sin[\varphi[xr]]) + a\,(xc - R\,Sin[\varphi[xr]])^2 == \left(2\,Fp + 4\,k\left(Rpin - \frac{x0}{2} + R\,Cos[\varphi[xr]] + yc[xc]\right)\right) yc'[xc]$$

$$\textbf{Eqn5} = \frac{\textbf{Eqn4[[1]]}}{\textbf{8 k}} \textbf{ == Expand}\left[\frac{\textbf{Eqn4[[2]]}}{\textbf{8 k}}\right]$$

$$\frac{c + b\,(xc - R\,Sin[\varphi[xr]]) + a\,(xc - R\,Sin[\varphi[xr]])^2}{8\,k} == \frac{Fp\,yc'[xc]}{4\,k} + \frac{1}{2}\,Rpin\,yc'[xc] - \frac{1}{4}\,x0\,yc'[xc] + \frac{1}{2}\,R\,Cos[\varphi[xr]]\,yc'[xc] + \frac{1}{2}\,yc[xc]\,yc'[xc]$$

$$\textbf{Eqn6 = Eqn5[[1]] - Eqn5[[2]][[1]] == Eqn5[[2]][[2]] + Eqn5[[2]][[3]] + Eqn5[[2]][[4]] + Eqn5[[2]][[5]]}$$

$$\frac{c + b\,(xc - R\,Sin[\varphi[xr]]) + a\,(xc - R\,Sin[\varphi[xr]])^2}{8\,k} - \frac{Fp\,yc'[xc]}{4\,k} == \frac{1}{2}\,Rpin\,yc'[xc] - \frac{1}{4}\,x0\,yc'[xc] + \frac{1}{2}\,R\,Cos[\varphi[xr]]\,yc'[xc] + \frac{1}{2}\,yc[xc]\,yc'[xc]$$

```
Eqn7 = -Eqn6[[1]] + Collect[Eqn6[[2]], yc'[xc]] ==
  0 (*Switch signs of each term to put in in standard form*)
```

$$-\frac{c + b\,(xc - R\,\mathrm{Sin}[\varphi[xr]]) + a\,(xc - R\,\mathrm{Sin}[\varphi[xr]])^2}{8\,k} + \frac{Fp\,yc'[xc]}{4\,k} + \left(\frac{Rpin}{2} - \frac{x0}{4} + \frac{1}{2}\,R\,\mathrm{Cos}[\varphi[xr]] + \frac{yc[xc]}{2}\right) yc'[xc] == 0$$

Corrected solution in same form as that published in ICRA 2005 paper

$$\mathbf{Eqn8 = Eqn7\ /.\ \left\{Sin[\varphi[xr]] \text{ -> } \frac{yc'[xc]}{\sqrt{1 + yc'[xc]^2}},\ Cos[\varphi[xr]] \text{ -> } \frac{1}{\sqrt{1 + yc'[xc]^2}}\right\}}$$

$$\frac{Fp\,yc'[xc]}{4\,k} + yc'[xc]\left(\frac{Rpin}{2} - \frac{x0}{4} + \frac{yc[xc]}{2} + \frac{R}{2\sqrt{1 + yc'[xc]^2}}\right) - \frac{c + b\left(xc - \frac{R\,yc'[xc]}{\sqrt{1+yc'[xc]^2}}\right) + a\left(xc - \frac{R\,yc'[xc]}{\sqrt{1+yc'[xc]^2}}\right)^2}{8\,k} == 0$$

Final simplified solution

```
Eqn10 = Expand[Eqn8[[1]] 8 k] == 0
```

$$-c - b\,xc - a\,xc^2 + 2\,Fp\,yc'[xc] + 4\,k\,Rpin\,yc'[xc] - 2\,k\,x0\,yc'[xc] + 4\,k\,yc[xc]\,yc'[xc] - \frac{a\,R^2\,yc'[xc]^2}{1 + yc'[xc]^2} + \frac{b\,R\,yc'[xc]}{\sqrt{1 + yc'[xc]^2}} + \frac{4\,k\,R\,yc'[xc]}{\sqrt{1 + yc'[xc]^2}} + \frac{2\,a\,R\,xc\,yc'[xc]}{\sqrt{1 + yc'[xc]^2}} == 0$$

```
Eqn11 = Eqn10[[1]][[1]] + Eqn10[[1]][[2]] + Eqn10[[1]][[3]] + Eqn10[[1]][[4]] +
    Eqn10[[1]][[5]] + Eqn10[[1]][[6]] + Eqn10[[1]][[7]] + Eqn10[[1]][[8]] ==
  Simplify[-(Eqn10[[1]][[9]] + Eqn10[[1]][[10]] + Eqn10[[1]][[11]])]
```

$$-c - b\,xc - a\,xc^2 + 2\,Fp\,yc'[xc] + 4\,k\,Rpin\,yc'[xc] - 2\,k\,x0\,yc'[xc] + 4\,k\,yc[xc]\,yc'[xc] - \frac{a\,R^2\,yc'[xc]^2}{1 + yc'[xc]^2} == -\frac{R\,(b + 4\,k + 2\,a\,xc)\,yc'[xc]}{\sqrt{1 + yc'[xc]^2}}$$

$$\mathbf{Eqn12 = Eqn11[[1]]\,\sqrt{(1 + yc'[xc]^2)}\ == Eqn11[[2]]\,\sqrt{(1 + yc'[xc]^2)}}$$

$$\sqrt{1 + yc'[xc]^2}\left(-c - b\,xc - a\,xc^2 + 2\,Fp\,yc'[xc] + 4\,k\,Rpin\,yc'[xc] - 2\,k\,x0\,yc'[xc] + 4\,k\,yc[xc]\,yc'[xc] - \frac{a\,R^2\,yc'[xc]^2}{1 + yc'[xc]^2}\right) == -R\,(b + 4\,k + 2\,a\,xc)\,yc'[xc]$$

```
Eqn13 = Apart[(Eqn12[[1]]^2 - Eqn12[[2]]^2) * (1 + yc'[xc]^2)] == 0
```

```
(c + b xc + a xc^2)^2 - 4 (c + b xc + a xc^2) (Fp + 2 k Rpin - k x0 + 2 k yc[xc]) yc'[xc] +
  (2 c^2 + 4 Fp^2 - b^2 R^2 + 2 a c R^2 - 8 b k R^2 - 16 k^2 R^2 + 16 Fp k Rpin + 16 k^2 Rpin^2 - 8 Fp k x0 -
     16 k^2 Rpin x0 + 4 k^2 x0^2 + 4 b c xc - 2 a b R^2 xc - 16 a k R^2 xc + 2 b^2 xc^2 + 4 a c xc^2 - 2 a^2 R^2 xc^2 +
     4 a b xc^3 + 2 a^2 xc^4 + 16 Fp k yc[xc] + 32 k^2 Rpin yc[xc] - 16 k^2 x0 yc[xc] + 16 k^2 yc[xc]^2)
   yc'[xc]^2 - 4 (2 c + a R^2 + 2 b xc + 2 a xc^2) (Fp + 2 k Rpin - k x0 + 2 k yc[xc]) yc'[xc]^3 +
  (c^2 + 8 Fp^2 - b^2 R^2 + 2 a c R^2 - 8 b k R^2 - 16 k^2 R^2 + a^2 R^4 + 32 Fp k Rpin + 32 k^2 Rpin^2 - 16 Fp k x0 -
     32 k^2 Rpin x0 + 8 k^2 x0^2 + 2 b c xc - 2 a b R^2 xc - 16 a k R^2 xc + b^2 xc^2 + 2 a c xc^2 - 2 a^2 R^2 xc^2 +
     2 a b xc^3 + a^2 xc^4 + 32 Fp k yc[xc] + 64 k^2 Rpin yc[xc] - 32 k^2 x0 yc[xc] + 32 k^2 yc[xc]^2)
   yc'[xc]^4 - 4 (c + a R^2 + b xc + a xc^2) (Fp + 2 k Rpin - k x0 + 2 k yc[xc]) yc'[xc]^5 +
  4 (Fp + 2 k Rpin - k x0 + 2 k yc[xc])^2 yc'[xc]^6 == 0
```

```
Eqn14 = Collect[ExpandAll[Eqn13[[1]]],
   {yc'[xc]^6, yc'[xc]^5, yc'[xc]^4, yc'[xc]^3, yc'[xc]^2, yc'[xc], yc[xc]}] == 0
```

```
c^2 + 2 b c xc + b^2 xc^2 + 2 a c xc^2 + 2 a b xc^3 + a^2 xc^4 +
  (-4 c Fp - 8 c k Rpin + 4 c k x0 - 4 b Fp xc - 8 b k Rpin xc + 4 b k x0 xc - 4 a Fp xc^2 -
     8 a k Rpin xc^2 + 4 a k x0 xc^2 + (-8 c k - 8 b k xc - 8 a k xc^2) yc[xc]) yc'[xc] +
  (2 c^2 + 4 Fp^2 - b^2 R^2 + 2 a c R^2 - 8 b k R^2 - 16 k^2 R^2 + 16 Fp k Rpin + 16 k^2 Rpin^2 - 8 Fp k x0 -
     16 k^2 Rpin x0 + 4 k^2 x0^2 + 4 b c xc - 2 a b R^2 xc - 16 a k R^2 xc + 2 b^2 xc^2 + 4 a c xc^2 - 2 a^2 R^2 xc^2 +
     4 a b xc^3 + 2 a^2 xc^4 + (16 Fp k + 32 k^2 Rpin - 16 k^2 x0) yc[xc] + 16 k^2 yc[xc]^2) yc'[xc]^2 +
  (-8 c Fp - 4 a Fp R^2 - 16 c k Rpin - 8 a k R^2 Rpin + 8 c k x0 + 4 a k R^2 x0 - 8 b Fp xc -
     16 b k Rpin xc + 8 b k x0 xc - 8 a Fp xc^2 - 16 a k Rpin xc^2 + 8 a k x0 xc^2 +
     (-16 c k - 8 a k R^2 - 16 b k xc - 16 a k xc^2) yc[xc]) yc'[xc]^3 +
  (c^2 + 8 Fp^2 - b^2 R^2 + 2 a c R^2 - 8 b k R^2 - 16 k^2 R^2 + a^2 R^4 + 32 Fp k Rpin + 32 k^2 Rpin^2 - 16 Fp k x0 -
     32 k^2 Rpin x0 + 8 k^2 x0^2 + 2 b c xc - 2 a b R^2 xc - 16 a k R^2 xc + b^2 xc^2 + 2 a c xc^2 - 2 a^2 R^2 xc^2 +
     2 a b xc^3 + a^2 xc^4 + (32 Fp k + 64 k^2 Rpin - 32 k^2 x0) yc[xc] + 32 k^2 yc[xc]^2) yc'[xc]^4 +
  (-4 c Fp - 4 a Fp R^2 - 8 c k Rpin - 8 a k R^2 Rpin + 4 c k x0 + 4 a k R^2 x0 - 4 b Fp xc -
     8 b k Rpin xc + 4 b k x0 xc - 4 a Fp xc^2 - 8 a k Rpin xc^2 + 4 a k x0 xc^2 +
     (-8 c k - 8 a k R^2 - 8 b k xc - 8 a k xc^2) yc[xc]) yc'[xc]^5 +
  (4 Fp^2 + 16 Fp k Rpin + 16 k^2 Rpin^2 - 8 Fp k x0 - 16 k^2 Rpin x0 + 4 k^2 x0^2 +
     (16 Fp k + 32 k^2 Rpin - 16 k^2 x0) yc[xc] + 16 k^2 yc[xc]^2) yc'[xc]^6 == 0
```

```
LbsPerInch2NewtonsPerCM = 1.7513;
Lbs2Newtons = 4.448;
Inches2CM = 2.54;

Rval = 0.25 * Inches2CM
Rpinval = 0.150 * Inches2CM
aval = 0.4;
bval = 0;
cval = 1;
kval = 1.46895 * LbsPerInch2NewtonsPerCM
x0val = 2.506 * Inches2CM
Fpval = 0.69 * Lbs2Newtons
```

```
0.635

0.381

2.57257

6.36524

3.06912
```

```
Eqn15 = Chop[Eqn14 /. {a → aval, b → bval, c → cval,
    k → kval, x0 → x0val, Fp → Fpval, R → Rval, Rpin → Rpinval} ]
```

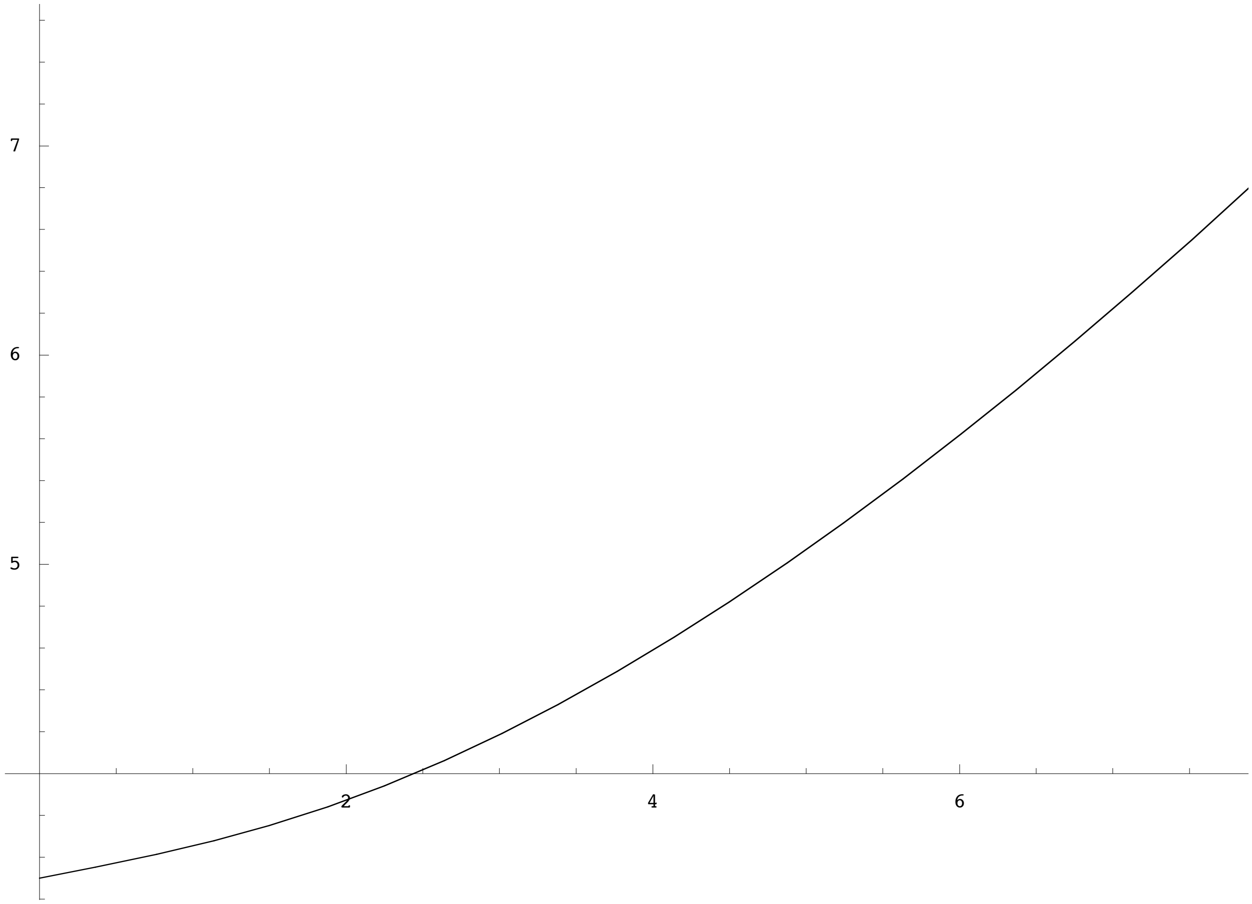
$$1 + 0.8\,xc^2 + 0.16\,xc^4 + (45.3825 + 18.153\,xc^2 + (-20.5806 - 8.23223\,xc^2)\,yc[xc])\,yc'[xc] +$$
$$(474.517 - 6.63888\,xc + 1.47097\,xc^2 + 0.32\,xc^4 - 466.999\,yc[xc] + 105.89\,yc[xc]^2)\,yc'[xc]^2 +$$
$$(98.0847 + 36.306\,xc^2 + (-44.4806 - 16.4645\,xc^2)\,yc[xc])\,yc'[xc]^3 +$$
$$(988.436 - 6.63888\,xc + 0.670968\,xc^2 + 0.16\,xc^4 - 933.998\,yc[xc] + 211.78\,yc[xc]^2)\,yc'[xc]^4 +$$
$$(52.7022 + 18.153\,xc^2 + (-23.9 - 8.23223\,xc^2)\,yc[xc])\,yc'[xc]^5 +$$
$$(514.892 - 466.999\,yc[xc] + 105.89\,yc[xc]^2)\,yc'[xc]^6 == 0$$

```
gg = NDSolve[{Eqn15 , yc[0] == 3.5}, yc[xc],
  {xc, 0, 9}, {MaxSteps → 100,  StartingStepSize → 0.01}]
```

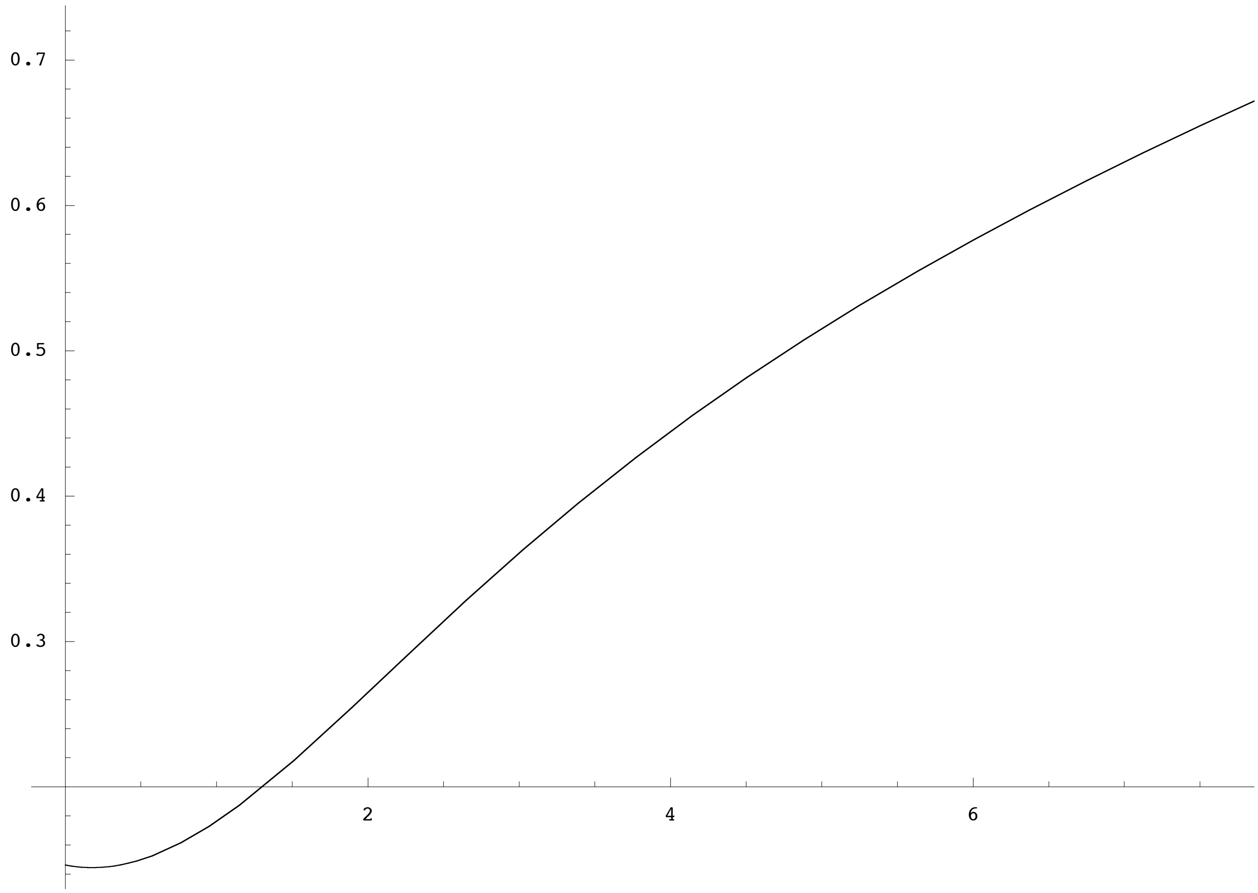
```
NDSolve::mxst : Maximum number of 100 steps reached at the point xc == 0.0000291866908687469`.

NDSolve::mxst : Maximum number of 100 steps reached at the point xc == 0.05572302877301485`.
```

```
{{yc[xc] → InterpolatingFunction[{{0., 9.}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 9.}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 0.0000291867}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 0.055723}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 9.}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 9.}}, <>][xc]}}
```

```
theG[xc_] = (yc[xc]) /. gg[[6]];
Plot[theG[xc], {xc, 0 , 9}]
Plot[Evaluate[∂x theG[x]], {x, 0, 9}]
```



- Graphics -



- Graphics -

```
profile = Table[theG[x], {x, 0, 8, 0.001}];
dprofile = Table[Evaluate[∂x theG[x]], {x, 0, 8, 0.001}];
Export["z:\\MastersDegree\\Mathematica\\new\\updated_profile.txt", profile, "Table"]
Export["z:\\MastersDegree\\Mathematica\\new\\updated_profile_deriv.txt",
 dprofile, "Table"]
```

General::spell1 : Possible spelling error: new symbol name "dprofile" is similar to existing symbol "profile".

z:\MastersDegree\Mathematica\new\updated_profile.txt

z:\MastersDegree\Mathematica\new\updated_profile_deriv.txt

Expanding Contour Spring Derivation

In[408]:=
```
Eqn1 = F[xr]/Fs[xr] == yc'[xc]
```

Out[408]= $\frac{F[xr]}{Fs[xr]} == yc'[xc]$

In[409]:=
```
Eqn2 = Eqn1 /. {F[xr] -> a xr^2 + b xr + c, Fs[xr] -> 4 k (yr[xr] - x0/2 + Rpin) + 2 Fp}
```

Out[409]= $\frac{c + b\,xr + a\,xr^2}{2\,Fp + 4\,k\,\left(Rpin - \frac{x0}{2} + yr[xr]\right)} == yc'[xc]$

In[410]:=
```
Eqn3 = Eqn2 /. {yr[xr] → yc[xc] + R Cos[φ[xr]]}
```

Out[410]= $\frac{c + b\,xr + a\,xr^2}{2\,Fp + 4\,k\,\left(Rpin - \frac{x0}{2} + R\,Cos[\varphi[xr]] + yc[xc]\right)} == yc'[xc]$

In[411]:=
```
Eqn4 =
  (Numerator[Eqn3[[1]]] /. {xr → xc - R Sin[φ[xr]]}) == Eqn3[[2]] Denominator[Eqn3[[1]]]
```

Out[411]= $c + b\,(xc - R\,Sin[\varphi[xr]]) + a\,(xc - R\,Sin[\varphi[xr]])^2 == \left(2\,Fp + 4\,k\,\left(Rpin - \frac{x0}{2} + R\,Cos[\varphi[xr]] + yc[xc]\right)\right)\,yc'[xc]$

In[412]:=
```
Eqn5 = Eqn4[[1]]/(8 k) == Expand[Eqn4[[2]]/(8 k)]
```

Out[412]= $\frac{c + b\,(xc - R\,Sin[\varphi[xr]]) + a\,(xc - R\,Sin[\varphi[xr]])^2}{8\,k} == \frac{Fp\,yc'[xc]}{4\,k} + \frac{1}{2}\,Rpin\,yc'[xc] - \frac{1}{4}\,x0\,yc'[xc] + \frac{1}{2}\,R\,Cos[\varphi[xr]]\,yc'[xc] + \frac{1}{2}\,yc[xc]\,yc'[xc]$

In[413]:=
```
Eqn6 = Eqn5[[1]] - Eqn5[[2]][[1]] ==
  Eqn5[[2]][[2]] + Eqn5[[2]][[3]] + Eqn5[[2]][[4]] + Eqn5[[2]][[5]]
```

Out[413]= $\frac{c + b\,(xc - R\,Sin[\varphi[xr]]) + a\,(xc - R\,Sin[\varphi[xr]])^2}{8\,k} - \frac{Fp\,yc'[xc]}{4\,k} == \frac{1}{2}\,Rpin\,yc'[xc] - \frac{1}{4}\,x0\,yc'[xc] + \frac{1}{2}\,R\,Cos[\varphi[xr]]\,yc'[xc] + \frac{1}{2}\,yc[xc]\,yc'[xc]$

```
Eqn7 = -Eqn6[[1]] + Collect[Eqn6[[2]], yc'[xc]] ==
  0 (*Switch signs of each term to put in in standard form*)
```

In[414]:= $$-\frac{c+b\,(xc-R\,Sin[\varphi[xr]])+a\,(xc-R\,Sin[\varphi[xr]])^2}{8\,k}+\frac{Fp\,yc'[xc]}{4\,k}+\left(\frac{Rpin}{2}-\frac{x0}{4}+\frac{1}{2}R\,Cos[\varphi[xr]]+\frac{yc[xc]}{2}\right)yc'[xc]==0$$

Out[414]= $$-\frac{c+b\,(xc-R\,Sin[\varphi[xr]])+a\,(xc-R\,Sin[\varphi[xr]])^2}{8\,k}+\frac{Fp\,yc'[xc]}{4\,k}+\left(\frac{Rpin}{2}-\frac{x0}{4}+\frac{1}{2}R\,Cos[\varphi[xr]]+\frac{yc[xc]}{2}\right)yc'[xc]==0$$

Corrected solution in same form as that

In[415]:= $$Eqn8 = Eqn7 \;/.\; \left\{Sin[\varphi[xr]] \to \frac{yc'[xc]}{\sqrt{1+yc'[xc]^2}},\; Cos[\varphi[xr]] \to \frac{1}{\sqrt{1+yc'[xc]^2}}\right\}$$

Out[415]= $$\frac{Fp\,yc'[xc]}{4\,k}+yc'[xc]\left(\frac{Rpin}{2}-\frac{x0}{4}+\frac{yc[xc]}{2}+\frac{R}{2\sqrt{1+yc'[xc]^2}}\right)-\frac{c+b\left(xc-\frac{R\,yc'[xc]}{\sqrt{1+yc'[xc]^2}}\right)+a\left(xc-\frac{R\,yc'[xc]}{\sqrt{1+yc'[xc]^2}}\right)^2}{8\,k}==0$$

Final simplified solution

In[416]:= $Eqn10 = Expand[Eqn8[[1]]\;8\,k] == 0$

Out[416]= $$-c-b\,xc-a\,xc^2+2\,Fp\,yc'[xc]+4\,k\,Rpin\,yc'[xc]-2\,k\,x0\,yc'[xc]+4\,k\,yc[xc]\,yc'[xc]-\frac{a\,R^2\,yc'[xc]^2}{1+yc'[xc]^2}+\frac{b\,R\,yc'[xc]}{\sqrt{1+yc'[xc]^2}}+\frac{4\,k\,R\,yc'[xc]}{\sqrt{1+yc'[xc]^2}}+\frac{2\,a\,R\,xc\,yc'[xc]}{\sqrt{1+yc'[xc]^2}}==0$$

In[417]:=
```
Eqn11 = Eqn10[[1]][[1]] + Eqn10[[1]][[2]] + Eqn10[[1]][[3]] + Eqn10[[1]][[4]] +
   Eqn10[[1]][[5]] + Eqn10[[1]][[6]] + Eqn10[[1]][[7]] + Eqn10[[1]][[8]] ==
  Simplify[-(Eqn10[[1]][[9]] + Eqn10[[1]][[10]] + Eqn10[[1]][[11]])]
```

Out[417]= $$-c-b\,xc-a\,xc^2+2\,Fp\,yc'[xc]+4\,k\,Rpin\,yc'[xc]-2\,k\,x0\,yc'[xc]+4\,k\,yc[xc]\,yc'[xc]-\frac{a\,R^2\,yc'[xc]^2}{1+yc'[xc]^2}==-\frac{R\,(b+4\,k+2\,a\,xc)\,yc'[xc]}{\sqrt{1+yc'[xc]^2}}$$

In[418]:= $Eqn12 = Eqn11[[1]]\,\sqrt{(1+yc'[xc]^2)} == Eqn11[[2]]\,\sqrt{(1+yc'[xc]^2)}$

Out[418]= $$\sqrt{1+yc'[xc]^2}\left(-c-b\,xc-a\,xc^2+2\,Fp\,yc'[xc]+4\,k\,Rpin\,yc'[xc]-2\,k\,x0\,yc'[xc]+4\,k\,yc[xc]\,yc'[xc]-\frac{a\,R^2\,yc'[xc]^2}{1+yc'[xc]^2}\right)==-R\,(b+4\,k+2\,a\,xc)\,yc'[xc]$$

In[419]:=
```
Eqn13 = Apart[(Eqn12[[1]]^2 - Eqn12[[2]]^2) * (1 + yc'[xc]^2)] == 0
```

Out[419]=
```
(c + b xc + a xc^2)^2 - 4 (c + b xc + a xc^2) (Fp + 2 k Rpin - k x0 + 2 k yc[xc]) yc'[xc] +
 (2 c^2 + 4 Fp^2 - b^2 R^2 + 2 a c R^2 - 8 b k R^2 - 16 k^2 R^2 + 16 Fp k Rpin + 16 k^2 Rpin^2 - 8 Fp k x0 -
    16 k^2 Rpin x0 + 4 k^2 x0^2 + 4 b c xc - 2 a b R^2 xc - 16 a k R^2 xc + 2 b^2 xc^2 + 4 a c xc^2 - 2 a^2 R^2 xc^2 +
    4 a b xc^3 + 2 a^2 xc^4 + 16 Fp k yc[xc] + 32 k^2 Rpin yc[xc] - 16 k^2 x0 yc[xc] + 16 k^2 yc[xc]^2)
  yc'[xc]^2 - 4 (2 c + a R^2 + 2 b xc + 2 a xc^2) (Fp + 2 k Rpin - k x0 + 2 k yc[xc]) yc'[xc]^3 +
 (c^2 + 8 Fp^2 - b^2 R^2 + 2 a c R^2 - 8 b k R^2 - 16 k^2 R^2 + a^2 R^4 + 32 Fp k Rpin + 32 k^2 Rpin^2 - 16 Fp k x0 -
    32 k^2 Rpin x0 + 8 k^2 x0^2 + 2 b c xc - 2 a b R^2 xc - 16 a k R^2 xc + b^2 xc^2 + 2 a c xc^2 - 2 a^2 R^2 xc^2 +
    2 a b xc^3 + a^2 xc^4 + 32 Fp k yc[xc] + 64 k^2 Rpin yc[xc] - 32 k^2 x0 yc[xc] + 32 k^2 yc[xc]^2)
  yc'[xc]^4 - 4 (c + a R^2 + b xc + a xc^2) (Fp + 2 k Rpin - k x0 + 2 k yc[xc]) yc'[xc]^5 +
 4 (Fp + 2 k Rpin - k x0 + 2 k yc[xc])^2 yc'[xc]^6 == 0
```

In[420]:=
```
Eqn14 = Collect[ExpandAll[Eqn13[[1]]],
   {yc'[xc]^6, yc'[xc]^5, yc'[xc]^4, yc'[xc]^3, yc'[xc]^2, yc'[xc], yc[xc]}] == 0
```

Out[420]=
```
c^2 + 2 b c xc + b^2 xc^2 + 2 a c xc^2 + 2 a b xc^3 + a^2 xc^4 +
 (-4 c Fp - 8 c k Rpin + 4 c k x0 - 4 b Fp xc - 8 b k Rpin xc + 4 b k x0 xc - 4 a Fp xc^2 -
    8 a k Rpin xc^2 + 4 a k x0 xc^2 + (-8 c k - 8 b k xc - 8 a k xc^2) yc[xc]) yc'[xc] +
 (2 c^2 + 4 Fp^2 - b^2 R^2 + 2 a c R^2 - 8 b k R^2 - 16 k^2 R^2 + 16 Fp k Rpin + 16 k^2 Rpin^2 - 8 Fp k x0 -
    16 k^2 Rpin x0 + 4 k^2 x0^2 + 4 b c xc - 2 a b R^2 xc - 16 a k R^2 xc + 2 b^2 xc^2 + 4 a c xc^2 - 2 a^2 R^2 xc^2 +
    4 a b xc^3 + 2 a^2 xc^4 + (16 Fp k + 32 k^2 Rpin - 16 k^2 x0) yc[xc] + 16 k^2 yc[xc]^2) yc'[xc]^2 +
 (-8 c Fp - 4 a Fp R^2 - 16 c k Rpin - 8 a k R^2 Rpin + 8 c k x0 + 4 a k R^2 x0 - 8 b Fp xc -
    16 b k Rpin xc + 8 b k x0 xc - 8 a Fp xc^2 - 16 a k Rpin xc^2 + 8 a k x0 xc^2 +
    (-16 c k - 8 a k R^2 - 16 b k xc - 16 a k xc^2) yc[xc]) yc'[xc]^3 +
 (c^2 + 8 Fp^2 - b^2 R^2 + 2 a c R^2 - 8 b k R^2 - 16 k^2 R^2 + a^2 R^4 + 32 Fp k Rpin + 32 k^2 Rpin^2 - 16 Fp k x0 -
    32 k^2 Rpin x0 + 8 k^2 x0^2 + 2 b c xc - 2 a b R^2 xc - 16 a k R^2 xc + b^2 xc^2 + 2 a c xc^2 - 2 a^2 R^2 xc^2 +
    2 a b xc^3 + a^2 xc^4 + (32 Fp k + 64 k^2 Rpin - 32 k^2 x0) yc[xc] + 32 k^2 yc[xc]^2) yc'[xc]^4 +
 (-4 c Fp - 4 a Fp R^2 - 8 c k Rpin - 8 a k R^2 Rpin + 4 c k x0 + 4 a k R^2 x0 - 4 b Fp xc -
    8 b k Rpin xc + 4 b k x0 xc - 4 a Fp xc^2 - 8 a k Rpin xc^2 + 4 a k x0 xc^2 +
    (-8 c k - 8 a k R^2 - 8 b k xc - 8 a k xc^2) yc[xc]) yc'[xc]^5 +
 (4 Fp^2 + 16 Fp k Rpin + 16 k^2 Rpin^2 - 8 Fp k x0 - 16 k^2 Rpin x0 + 4 k^2 x0^2 +
    (16 Fp k + 32 k^2 Rpin - 16 k^2 x0) yc[xc] + 16 k^2 yc[xc]^2) yc'[xc]^6 == 0
```

In[421]:=
```
LbsPerInch2NewtonsPerCM = 1.7513;
Lbs2Newtons = 4.448;
Inches2CM = 2.54;

Rval = 0.25 * Inches2CM
Rpinval = 0.150 * Inches2CM
aval = 0.4;
bval = 0;
cval = 1;
kval = 1.46895 * LbsPerInch2NewtonsPerCM
x0val = 2.506 * Inches2CM
Fpval = 0.69 * Lbs2Newtons
```

Out[424]= 0.635

Out[425]= 0.381

Out[429]= 2.57257

Out[430]= 6.36524

Out[431]= 3.06912

In[432]:=
```
Eqn15 = Chop[Eqn14 /. {a → aval, b → bval, c → cval,
     k → kval, x0 → x0val, Fp → Fpval, R → Rval, Rpin → Rpinval} ]
```

Out[432]=
```
1. + 0.8 xc^2 + 0.16 xc^4 + (45.3825 + 18.153 xc^2 + (-20.5806 - 8.23223 xc^2) yc[xc]) yc′[xc] +
  (474.517 - 6.63888 xc + 1.47097 xc^2 + 0.32 xc^4 - 466.999 yc[xc] + 105.89 yc[xc]^2) yc′[xc]^2 +
  (98.0847 + 36.306 xc^2 + (-44.4806 - 16.4645 xc^2) yc[xc]) yc′[xc]^3 +
  (988.436 - 6.63888 xc + 0.670968 xc^2 + 0.16 xc^4 - 933.998 yc[xc] + 211.78 yc[xc]^2) yc′[xc]^4 +
  (52.7022 + 18.153 xc^2 + (-23.9 - 8.23223 xc^2) yc[xc]) yc′[xc]^5 +
  (514.892 - 466.999 yc[xc] + 105.89 yc[xc]^2) yc′[xc]^6 == 0
```

In[435]:=
```
gg = NDSolve[{Eqn15 , yc[0] == 3.5}, yc[xc],
   {xc, 0, 9}, {MaxSteps → 100,  StartingStepSize → 0.01}]
```

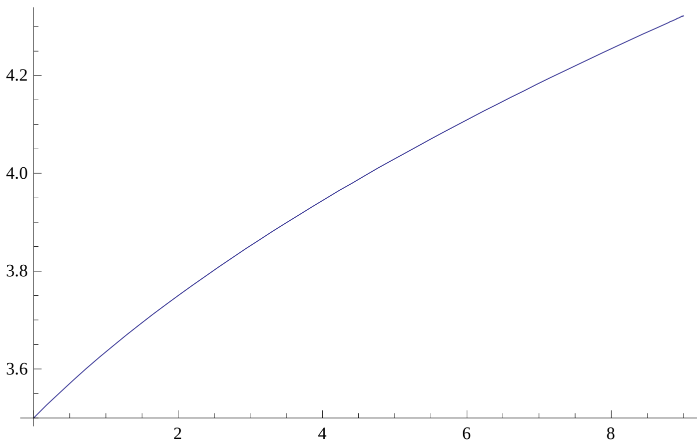
NDSolve::mxst : Maximum number of 100 steps reached at the point xc == 0.3943667775886695`. ≫

NDSolve::mxst : Maximum number of 100 steps reached at the point xc == 0.39711107818703134`. ≫

Out[435]=
```
{{yc[xc] → InterpolatingFunction[{{0., 9.}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 9.}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 0.394367}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 0.397111}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 9.}}, <>][xc]},
 {yc[xc] → InterpolatingFunction[{{0., 9.}}, <>][xc]}}
```

In[457]:=
```
theG[xc_] = (yc[xc]) /. gg[[6]];
Plot[theG[xc], {xc, 0 , 9}]
Plot[Evaluate[∂x theG[x]], {x, 0, 9}]
```

Out[458]=



Out[459]=